

Field Stars, Open Clusters, and the Galactic Abundance Gradient

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ABSTRACT

The effects of the orbital diffusion of stars and clusters on the galactic abundance gradient have been modelled. For a single generation of field stars, the results confirm previous analyses based upon the assumption of a linear gradient with galactocentric distance. Though the dispersion in metallicity at a given radius grows with time, the underlying slope across the disk remains unchanged. For a disk dominated by a discontinuity in $[\text{Fe}/\text{H}]$ beyond the solar circle, but no gradient within the inner or outer disk, a gradient is derived for any sample whose galactocentric baseline overlaps the discontinuity. The narrower the range in distance around the discontinuity, the steeper the gradient. The consequence of orbital diffusion is to shift the observable edge of the transition in $[\text{Fe}/\text{H}]$ toward smaller galactocentric distance, enhancing the detection of a gradient. In contrast, star clusters of a single generation evaporate so quickly that the small surviving sample of the oldest clusters is statistically incapable of defining any gradient except a linear case. Inclusion of multiple generations of stars and clusters with a plausible age-metallicity relation enhances these patterns by expanding the metallicity range at a given galactocentric distance through the inclusion of stars with a wide range in both age and galactocentric origin. For field stars, the metallicity distribution of stars in the solar neighborhood is virtually indistinguishable for the two cases modelled. Because ages, abundances, and distances can be reliably determined for clusters and any sample of clusters is invariably dominated by those clusters formed over the last 2 Gyrs, the cluster population offers a viable means for finding detailed structure within the recent galactic abundance gradient. Using clusters to extend the galactocentric baseline would supply an additional critical test of the discontinuity.

Subject headings: Galaxy: evolution – Galaxy: disk – Galaxy: kinematics and dynamics

1. INTRODUCTION

The fundamental goal of Galactic stellar populations is an understanding of the chemical and dynamical history of the Milky Way. Since the seminal work of Eggen et al. (1962), the implicit assumption that kinematics, composition, location, and age were correlated has gradually given way to a more chaotic view where modest trends among these parameters are subsumed beneath a large dispersion in each as a function of time. Though a fragmented origin for the Milky Way is most commonly associated with the halo, as first suggested by Searle & Zinn (1978), the need for a greater degree of randomness within the populations has been pushed toward more recent galactic epochs,

in part, by the evidence for and analyses of the thick disk population (Gilmore & Reid 1983) and the field star analysis of the solar neighborhood by Edvardsson et al. (1993).

The work of Edvardsson et al. (1993) has taken on particular significance because of its implication that the metallicity of stars at a given age covers a range of at least a factor of four and possibly as large as a factor of eight, significantly larger than found in earlier analyses of solar neighborhood stars (Twarog 1980; Meusinger et al. 1991). Paradoxically, the increased scatter appeared in a sample which had supposedly more reliable ages and abundances, while reproducing the same overall mean trends with age as the earlier studies.

Though Edvardsson et al. (1993) explicitly stated that their sample was biased and should not be used to analyze the age-metallicity history of the disk, their own analysis and discussion of the data implied that the bias was modest and that the metallicity spread was real. The bias originated in the sample selection. The distribution of $[\text{Fe}/\text{H}]$ among the bright F stars has long been known to exhibit a paucity of stars with $[\text{Fe}/\text{H}]$ below -0.4 , supposedly the metallicity range populated predominantly by the older stars of the disk. Since the goals of Edvardsson et al. (1993) extended beyond the delineation of the age-metallicity relation to any metallicity-based trends, the apparent-magnitude-limited sample was expanded to include a significant number of F stars with lower $[\text{Fe}/\text{H}]$ to ensure an approximately uniform distribution between $[\text{Fe}/\text{H}] = +0.2$ and -0.9 . Fortunately, with proper weighting of the sample to correct for the distortion, the bias can be and was, in theory, accounted for. The other potential source of sample distortion, orbital diffusion, was supposedly eliminated by the kinematic analysis, i.e., the stars of a given age that formed within an annulus of modest width centered on the sun exhibited the same degree of abundance scatter as the overall sample. Attempts to explain the degree of inhomogeneity within the context of galactic evolution have been numerous (Wielen et al. 1996; Pilyugin & Edmunds 1996; van den Hoek & de Jong 1997).

Beyond the lack of an explanation for the absence of comparable scatter in earlier field star analyses of an unbiased sample, the conclusions of Edvardsson et al. (1993) are contradicted by two observational constraints from a larger scale than the solar neighborhood:

(a) Studies of HII regions across the galactic disk indicate that at a given galactocentric distance, the dispersion in $[\text{m}/\text{H}]$ is on the order of ± 0.1 dex, significantly smaller than found within the field star sample, though there is evidence of a gradient in $[\text{m}/\text{H}]$ of typically between -0.05 and -0.10 dex/kpc (see and references therein) HW99.

(b) Twarog et al. (1997) have shown that the open clusters within galactocentric distance (R_{GC}) of 10 kpc (R_{GC} for the sun is 8.5 kpc), when placed on a uniform metallicity scale, have a mean $[\text{Fe}/\text{H}] \sim 0.0$ with a dispersion of only ± 0.10 dex. This result occurs whether one uses all the clusters

or separates them into young and old (age greater than 1.5 Gyr). No cluster within $R_{GC} = 10$ kpc has a reliably determined $[\text{Fe}/\text{H}]$ below -0.2 , while the majority of the field star sample, adjusted for selection effects, populates this regime.

There are a variety of ways of explaining these discrepancies. The HII regions could be discarded because they represent the current state of the interstellar medium. They would be irrelevant if the chemical homogeneity of the gas within the disk at a given galactocentric distance today is not typical of the dispersion in $[\text{Fe}/\text{H}]$ 2 to 10 Gyrs ago or if the instantaneous dispersion is not representative of the spread after a few Gyrs worth of differential galactic rotation. The former explanation implies that we somehow live at a special time in the history of the galactic disk and is contradicted by the open cluster data to an age of approximately 5 Gyr. The latter interpretation implies some form of orbital diffusion. Stars formed at different galactocentric distances, within the context of a radially-dependent $[\text{Fe}/\text{H}]$, are dispersed with time over increasingly larger R_{GC} , turning the range in the abundance with R_{GC} into a spread in $[\text{Fe}/\text{H}]$ at a given R_{GC} (Wielen et al. 1996). This interpretation is challenged by the kinematic analysis of Edvardsson et al. (1993) discussed above and by the open cluster data which exhibit little to no evidence for a gradient in $[\text{Fe}/\text{H}]$ between $R_{GC} = 6$ and 10 kpc. There is, however, a significant discontinuity in $[\text{Fe}/\text{H}]$ for clusters beyond $R_{GC} = 10$ kpc; the mean abundance for the modest sample beyond this transition is $[\text{Fe}/\text{H}] \sim -0.3$.

For the open cluster sample, the only objection possible is the rather artificial one that the sample is unrepresentative of the field star population. Either the open clusters do not form with the same abundance distribution as the field stars or the metal-poor portion of the cluster sample is preferentially destroyed. Any attempt to use abundance errors to explain the result is contradicted by the care used in selecting, analyzing, and combining the observational data, as well as the simple point that the dispersion of ± 0.10 in $[\text{Fe}/\text{H}]$ is an upper limit to the intrinsic spread. If significant abundance errors or an abundance gradient exist within the sample, the estimated intrinsic dispersion, corrected for these effects, should decrease, making the disagreement with the field star sample larger.

Two related papers have attempted to address

this issue. Garnett & Kobulnicky (2000) have presented an analysis of the Edvardsson et al. (1993) data sorted by distance, i.e., separated into two groups divided by the distance boundary of 30 pc from the sun. The investigation clearly demonstrates that the more distant sample differs significantly from the nearby sample in kinematics, metallicity distribution, and, most important, age-metallicity relation. Though the more distant sample covers the same range in age as the nearby stars, the average $[\text{Fe}/\text{H}]$ is lower in the mean at each age, exhibiting a gradual increase in $[\text{Fe}/\text{H}]$ with time. In sharp contrast, the nearby sample shows a rapid rise in $[\text{Fe}/\text{H}]$ among the older stars before leveling off among the younger stars. Garnett & Kobulnicky (2000) conclude that this difference reflects the bias in the sample of Edvardsson et al. (1993) toward stars with lower than average $[\text{Fe}/\text{H}]$, a bias most easily demonstrated in a distance-sorted sample because of the need to go to a larger volume to obtain a uniform distribution in $[\text{Fe}/\text{H}]$, the point noted above.

The weak point in the discussion, however, is the implication that if an unbiased sample had been chosen the dispersion in $[\text{Fe}/\text{H}]$ would reduce to a value comparable to that of the clusters and HII regions, assuming no additional external effects are in operation. Edvardsson et al. (1993) understood the issue and attempted to correct for it by applying a metallicity-based volume correction. The adjustment produced only a minor reduction in the dispersion with age. Thus, either the corrections were grossly inadequate or an additional source must be found for the dispersion. It should be noted that in the three of the four primary analyses of the age-metallicity relation to date, Twarog (1980), Meusinger et al. (1991), and Edvardsson et al. (1993), the stars with ages below 2 Gyr exhibited the smallest dispersion in $[\text{Fe}/\text{H}]$, while the stars between 4 and 10 Gyr are approximately constant. The size of the average dispersion, however, has grown larger with each subsequent analysis. The one exception to this trend is the chromospheric-age-based analysis by Rocha-Pinto et al. (2000) of 552 field F and G dwarfs. Using an age scale tied to the system of Edvardsson et al. (1993), Rocha-Pinto et al. (2000) derive an age-metallicity relation with a typical scatter of 0.13 dex, virtually identical to that found by Twarog (1980), corroborating

the cluster result of Twarog et al. (1997) and the claim by Garnett & Kobulnicky (2000) that some subtle form of selection bias dominates the Edvardsson et al. (1993) sample. A level of concern, however, is raised by the fact that the age-metallicity relation derived by Rocha-Pinto et al. (2000) shows a linear growth over time, the same result found by Edvardsson et al. (1993) using a supposedly biased sample. This simple trend is contradicted by the analysis of the nearby sample of Edvardsson et al. (1993) in Garnett & Kobulnicky (2000) and the analysis by Meusinger et al. (1991) of the unbiased sample of Twarog (1980) using isochrones of the same vintage as used by Edvardsson et al. (1993). Moreover, it leads to a metallicity for the recently-formed stars in the disk which is comparable to the Hyades, in contradiction with both young clusters and HII region abundances. Thus, though we have come full circle in our view of a well-defined change in the mean metallicity of the disk over time, the exact shape of the curve remains controversial.

If the disk stars with $[\text{Fe}/\text{H}]$ below -0.3 are not a statistical fluke in an otherwise homogeneous solar neighborhood, what are their origins? The discussion of Garnett & Kobulnicky (2000) suggests that part of the problem may arise from contamination of the sample by stars of the thick disk, irrespective of its origin. This option, however, is eliminated by the second paper in the pair, the kinematic analysis of the Edvardsson et al. (1993) data by Quillen & Garnett (2000), where it is found that the velocity dispersion of the disk rises among stars in the age range of 0 to 3 Gyr before leveling off between 3 and 9 Gyrs. The thick disk makes its appearance in the form of an abrupt increase in velocity dispersion among stars approximately 10 Gyrs and older. What is important about this result is that it identifies the thick disk as the product of a specific event within a narrow range of time. The stars which populate this event are readily identified in the sample of Edvardsson et al. (1993); they enter the analysis almost exclusively based upon their abundance and typically have $[\text{Fe}/\text{H}] = -0.5$ or less. If correct, these stars and this event have no relevance to the origin of the stars between 2 and 9 Gyrs of age with $[\text{Fe}/\text{H}]$ between -0.2 and -0.5 , the source of the excess dispersion in $[\text{Fe}/\text{H}]$ relative to the clusters and the HII regions.

The purpose of this paper is to test the possibility that the cluster data and the field star data are, in fact, compatible and that the results of Edvardsson et al. (1993) reflect a combination of both selection bias and stellar orbital diffusion. In this scenario, the homogeneity of the cluster sample over at least half the lifetime of the disk is maintained by the lack of a significant gradient coupled with a cluster dissolution timescale shorter than the diffusion timescale within the galactic plane. In contrast, the field stars can be identified at any age. Orbital diffusion of stars formed from beyond the break in $[\text{Fe}/\text{H}]$ at $R_{GC} = 10$ kpc, coupled with the age-metallicity relation, readily explains the inhomogeneity found within the solar neighborhood.

Section 2 will outline the approach to modelling the effects of diffusion and test the basic program by comparison to previous work, Sec. 3 details the impact of cluster destruction, Sec. 4 combines the kinematic and cluster effects with the chemical history of the disk to model the cumulative sample found within the solar neighborhood, and Sec. 5 discusses the implications of the simulations.

2. The Model: Background

The fact that the kinematics of stars within the galactic disk in the neighborhood of the sun change with time of formation over the life of the disk is unquestioned. Decades of work have repeatedly shown that the dispersions in U , V , and W , the stellar orbital velocity components relative to the local standard of rest, increase as the mean age of the stellar sample increases. The primary points of contention are linked to the details:

(a) What are the exact trends in the dispersions with time?

(b) What physical mechanisms/interactions are the source of the velocity variation?

(c) What are the relative contributions of the intrinsic dispersion in the gas from which the stars form and the long term heating of the galactic disk to the combined velocity trend?

(d) Does the average galactocentric distance of a star's orbit undergo significant changes over time, irrespective of the mechanism which leads to the growth in the velocity dispersion?

Clearly, all of these issues are coupled and the extent to which one accepts or rejects the plausi-

bility of orbital diffusion over extensive ranges in galactocentric distance, point (d) above, is dependent upon how one answers (a) through (c).

The approach adopted here is that developed by Wielen (1977), but expanded upon and applied over the last two decades by Wielen and his coworkers (Wielen & Fuchs 1985, 1988; Wielen et al. 1992; Just et al. 1996; Wielen et al. 1996); readers are encouraged to investigate these papers, particularly Wielen (1977), for detailed discussions of the underlying derivations. In short form, stars are assumed to orbit an axially symmetric galactic potential along paths which are the superposition of a small ellipse and a circle. The epicyclic approximation (Lindblad 1959) has the ellipse centered upon a point, the local standard of rest, which moves at constant speed in a circle about the galactic center. The star moves about the ellipse whose size and shape are defined by the degree of non-circularity inherent in the galactic orbit due to the deviations of the star's velocity from that of the local standard of rest.

The fact that the observed velocity dispersions in all three velocity components relative to the local standard of rest grow with the mean age of the sample implies that components of the potential must exist beyond the combination of an axially symmetric galactic potential and the effects of radial variations in the potential. Some additional acceleration process or processes must exist to perturb the orbits over time. Since star-star interactions are hopelessly inefficient within the disk, most attempts to resolve the issue have been constructed about stellar interactions with larger scale fluctuations in the potential, specifically molecular clouds (Spitzer & Schwarzschild 1953) or density waves (Barbanis & Woltjer 1967). Since their initial introduction into the issue, both forms of the solution have been tested and come up short in their ability to generate an effect of the required size, though Jenkins (1992) has demonstrated that a combination of some form of both processes may do the trick.

The implicit assumption of the diffusion approach is that while knowing the exact nature of the mechanism is a valuable check on the reality of the perturbations, from a statistical standpoint it is a detail which isn't crucial to modelling stellar kinematics. The acceleration process can be approximated by a sequence of stochastic perturba-

tions of short duration; the perturbation changes the velocity of the star relative to the local standard of rest, but not the position. While the average change in velocity is zero, the average of the sum of the squares of the changes in velocity grows with time. The rate of growth of the dispersion with time is generated by a diffusion coefficient which may take any form with time. The mathematical form of the coefficient is defined empirically; the product of the perturbations over time must reproduce the observed changes in velocity dispersion over time. The link between the diffusion coefficient and the velocity dispersions is made by calculating the differences in position and velocity over time between the stellar distribution with diffusion and in the absence of diffusion using a set of linearized differential equations which include the effects of random perturbations in the three velocity components.

In his original study, Wielen (1977) investigated three functional forms for the diffusion coefficient: (a) constant with time, (b) constant with time but inversely dependent upon the peculiar velocity of the star, and (c) inversely proportional to the peculiar velocity of the star and, on average, declining exponentially with time. Within the large uncertainties imposed by the observational data, all three functions with an appropriate choice of coefficients could reproduce the observations. In general, the dispersions grew at the slowest pace for case (b), while changing most rapidly among the oldest stars for case (c). The qualitative conclusion was same for all three cases. In reproducing the trend in velocity dispersion with age via stochastic perturbations, within 1 to 2 Gyr the average star will have its orbit altered so much that any attempt to backtrack the star's orbit to its origin within the Galaxy using its current position and kinematics is futile. Quantitatively, all three cases showed significant changes in velocity early on, but case (b) implied a levelling off of the growth in the dispersion as the sample aged.

Subsequent work has focused on a variety of issues including the size of the diffusion in the radial direction from the galactic center which implied a significant change in both the size of the epicyclic ellipse and the instantaneous galactocentric distance of the center of the ellipse. The importance of the latter issue is that analysis of the solar neighborhood sample is often based upon

the assumption that calculation of the mean orbital distance of a star using the instantaneous velocities provides a direct estimate of the galactocentric location at the time of formation (see, e.g.,]E93. If radial diffusion is as large as Wielen's approach implies, alternate means of determining the location of origin, such as combining an assumed abundance gradient with age (Wielen et al. 1996), must be employed.

Because the size of the diffusion coefficient is determined empirically by optimizing the match of the model to the observed velocity dispersions over time, much of the criticism of Wielen's work has centered on the claim that the derived observational trend with age is exaggerated. Both Freeman (1991) and Quillen & Garnett (2000) have analyzed the data of Edvardsson et al. (1993) and concluded that the velocity dispersions only grow with increasing age to 3 Gyr, then the effect saturates. If true, the diffusion process would be irrelevant beyond this point and any form of radial diffusion would be limited to the range imposed by a 3 Gyr time frame.

The criticisms of Wielen's approach are weak on two counts. First, though the models with constant diffusion over time do exhibit significant growth in the velocity dispersions among the oldest stars, the model with a time-constant but velocity-dependent coefficient leads to a rapid rise in velocity dispersion early on, followed by a gradual flattening of the curve at increasing age. Despite this declining effect, radial diffusion remains significant for this model. Second, the claim that the velocity trend with age remains flat beyond 3 Gyr is not supported by other analyses of the solar neighborhood, both prior to Edvardsson et al. (1993) and, most recently, by Fuchs et al. (2000) of *Hipparcos* data. This question is often framed in terms of the power-law relationship between the total velocity dispersion and time. The original analysis by Wielen (1977) implied a dispersion which varied as $t^{0.5}$. A more recent study by Binney et al. (2000) has derived a power near 0.33, but Fuchs et al. (2000) have shown that the discrepancy with 0.5 is mild and that all the modern data sets are consistent within the errors with an increasing velocity dispersion over time at a rate similar to that originally derived by Wielen (1977). Further resolution of this question may have to wait for even larger samples of reliable

kinematic information.

2.1. The Model: Galactic Parameters

Though our primary interest is with the stars within 2 to 3 kpc of the sun, we have constructed our disk distribution over a galactocentric range from 2 to 20 kpc, with the sun located at 8.5 kpc. Clearly, the reliability of the analysis declines with increasing distance from the sun, but the effects of diffusion may lead to drifts over distances of a few kpc, necessitating at least an approximate attempt at including the effects caused by stars and clusters well beyond the area of interest. The standard model begins with 1000 point masses assigned coordinates in a cylindrical frame centered on the galactic center. The longitudinal coordinate in the plane, θ , is assigned randomly between 0° and 360° . The radial coordinate is also assigned in random fashion, but weighted to ensure that the radial surface density of points produces an exponential variation with galactocentric distance, typical of what is observed for spiral galaxies and consistent with the parameters for the luminosity profile of the Milky Way. For a simple overview of the properties of the Milky Way and galaxies in general, the reader is referred to the summary in Chapter 1 of Binney & Tremaine (1987); a more extensive discussion can be found in Binney & Merrifield (1998).

As a simplistic approximation, the rotation curve of the galaxy is assumed to be flat over the range from 2 to 20 kpc, implying that the Oort constants must be set such that $A = -B$. The value adopted for the rotation curve is $V_{rot} = 220$ km/sec. The sample of stars is initially assumed to form with an average velocity of 0 relative to the local standard of rest (purely circular orbital velocity about the galactic center) at each galactocentric distance, but individual stars have velocities in all three components which are distributed in gaussian fashion about the mean. The initial gaussian dispersions at $R_{GC} = 8.5$ kpc are selected to resemble the velocity dispersions found for recently formed stars in the solar neighborhood. Away from the sun, the velocity dispersion in the W component is assumed to follow an exponential trend with galactocentric distance with an e-folding length equal to twice that of the disk (Bottenga 1993). Given the relationship between the Oort constants set by the flat rotation

curve, the initial velocity dispersion in U becomes a scaled value (1.82) of σ_W and the dispersion in V is a scaled value (0.71) of the dispersion in U .

Given U , V , W , and R , the galactocentric distance, the star's initial epicyclic orbit is derived. Each star's motion is followed for two distinct cases: (a) assuming no perturbations by outside forces and (b) assuming that perturbations are administered to all three velocity components with randomly varying strength at regular time intervals (10^7 yrs). The size of the perturbations is chosen to ensure a cumulative gaussian distribution at a given time, with the dispersion in the gaussian fixed by the value of the diffusion coefficient. As discussed earlier, the diffusion coefficient may be defined as a constant, as a time-varying function, or as a time-varying function of the velocity of the star. Whichever function is adopted, the diffusion parameters are selected to reproduce the empirically defined trends in σ over time; for this reason, the timescale between perturbations is not a factor and has been chosen to ensure a statistically smooth cumulative alteration in the velocity components on a timescale comparable to the galactic orbital period of the sun, about 0.2 Gyrs. Because of the way the differential equations are constructed, one can readily calculate the positional and kinematic differences between the unperturbed orbit and the perturbed orbit over time (Wielen 1977).

Though we have modelled the same parameterizations of the diffusion coefficient as Wielen (1977), we will only discuss the results for the case where the coefficient is constant with time but inversely proportional to the velocity since this case had the smallest long-term diffusion effect on the orbits. Fig. 1 demonstrates the trend over time for the dispersions in total velocity and each of the three components averaged from a set of 10 program runs. Though the points illustrate the dispersions for the entire sample over all galactocentric distances, there is no statistically significant difference compared to the trend if one isolates the stars within 1.5 kpc of the sun. As expected from previous work, for older stars the velocity dispersion approaches a relation proportional to $t^{0.5}$.

2.2. The Model Abundance Gradient and Metallicity Distribution

Though the Galaxy represents a composite system constructed of multiple generations of stars, to gain some insight into the way in which the disk distribution of stars is affected by the cumulative perturbations, we first analyze the temporal and spatial evolution of a single generation of stars over a time frame of 10 Gyrs. Our focus is on the degree to which the galactic abundance gradient is altered over time, both in shape and dispersion, and the time variation of the metallicity distribution of stars found within the solar neighborhood, where we artificially classify the solar neighborhood as the region within 1.5 kpc of the sun, or the zone between $R_{GC} = 7$ and 10 kpc.

For the galactic abundance gradient we select two options. The first case is the traditional linear gradient with a slope of -0.07 dex/kpc (see and the discussion therein]TAT97. The zero-point of this scale is a major concern. If we use the clusters inside $R_{GC} = 10$ kpc, the mean $[\text{Fe}/\text{H}]$ for 62 clusters in Twarog et al. (1997) is solar. Given their age distribution, the clusters in this sample should sample primarily the disk in the solar neighborhood over the last 2 Gyrs. Unless the metallicity of the disk has declined in the recent past, the recently formed stars in the solar neighborhood should have at least this $[\text{Fe}/\text{H}]$. The problem is that if we extrapolate the gradient to the galactic center, the average abundance of stars there should be $[\text{Fe}/\text{H}] \sim +0.6$. The stellar population near the galactic center has an extensive but contradictory history. Originally believed to be super-metal-rich (Whitford & Rich 1983; Rich 1988; Frogel et al. 1990; Terndrup et al. 1991; Geisler & Friel 1992), more recent work (McWilliam & Rich 1994; Ramirez et al. 2000; Carr et al. 2000) has produced abundances comparable to solar. The complicating issue is the difficulty of separating true disk stars from the bulge population but, to date, no evidence exists for any significant population of super-metal-rich stars near the galactic center, bulge or otherwise.

To guarantee that the stars near the galactic center have a plausible metallicity, we set the mean $[\text{Fe}/\text{H}]$ in the solar neighborhood at -0.2 , emphasizing again that this is inconsistent with the data for both young clusters and recently formed stars.

The simplest solution to the contradiction is that the gradient changes slope and flattens somewhere within $R_{GC} = 10$ kpc. Theoretical chemical evolution models attempting to produce galactic abundance gradients produce such flattening as a natural consequence of the more rapid evolution of the inner disk, i.e., the disk develops from the inside out. However, while evidence exists for flatter gradients among some elements within 4 kpc of the galactic center, the sizes of the samples are inadequate to accept this as a definitive observational constraint (see and the discussion therein]HP00.

The flattening of the abundance gradient is an implicit assumption of the second option. For this we assume that there is no gradient within $R_{GC} = 10$ kpc or beyond this galactocentric distance. The mean metallicity in the inner zone is set to $[\text{Fe}/\text{H}] = 0.00$, while the mean $[\text{Fe}/\text{H}]$ of the outer zone is -0.3 . This creates a discontinuity of -0.3 dex at $R_{GC} = 10$ kpc. Note that the step function is artificially sharp; it may well be that there are modest (-0.02 dex/kpc) gradients in both regions and the discontinuity extends over 0.5 kpc. Such distinctions are significant at a level of interest well below the limits set by the current observational data and minor compared to the impact of other modelling assumptions.

In both cases, it is assumed that the stars form at a given galactocentric distance with a gaussian metallicity distribution and a dispersion of 0.1 dex, typical of both the cluster sample (Twarog et al. 1997) and the abundance spread among HII regions (Henry & Worthey 1999), though larger values of 0.2 and 0.3 dex have been tested. Fig. 2 illustrates the evolution of the abundance gradient sampled at t equals 0 Gyrs, after 5 Gyrs of evolution, and after 10 Gyrs for (a) the linear case and (b) the discontinuity. For the linear case, there is only weak evidence for change in the slope of the gradient, though the scatter at a given R clearly grows as stars drift away from their initial location. From multiple runs of the model, the abundance gradient derived from data between $R = 5$ and 15 kpc changes from the initial value of -0.070 to -0.061 ± 0.003 . In contrast, the dispersion in $[\text{Fe}/\text{H}]$ grows significantly from an initial value of 0.10 dex to 0.154 ± 0.006 dex.

The case for the discontinuity is somewhat more of a challenge to interpret. As expected, for re-

gions well away from the discontinuity, the abundance gradients remain flat with little effect on the dispersion since all the stars in the surrounding neighborhood have similar abundances. However, as one studies the disk closer to the break, as the disk ages, diffusion moves an increasingly larger fraction of stars across the boundary in both directions. The impact is to create an annular zone where the metallicity spread approaches a superposition of two gaussians. The greater the passage of time, the wider the zone where this mixture exists.

To achieve some sense of what this means for the abundance gradient, we have derived the gradient in the same way noted above. A linear relation was fit to all the data between 5 and 15 kpc. Over this range, the initial slope was found to be -0.042 ± 0.002 , becoming shallower to -0.034 ± 0.002 by 10 Gyrs. Obviously, the slope is strongly dependent upon the distance range used to define it; if we had chosen 7.5 to 12.5 kpc, the derived gradient would be similar to the observed value of -0.07 . For the dispersion, the initial spread is typically 0.122 ± 0.002 dex and increases only slightly to 0.136 ± 0.003 . The very modest change is readily explained by the fact that if one uses the entire range between 5 and 15 kpc, the relative contribution of stars from either side of the discontinuity remains almost unchanged; the stars mostly shift position within this annulus.

To get a handle on the importance of diffusion in this model, we need to narrow the annular range to a more practical value and to move off center from the breakpoint. We do this by defining the zone between 7 and 10 kpc as the solar neighborhood and deriving the metallicity distribution within this strip. The results are illustrated in Fig. 3, where a and b refer to the same gradient models as in Fig. 2. For the linear case, the distribution is initially a gaussian with a slightly broader dispersion than 0.10 dex because of the range of galactocentric distance from which the sample is drawn. As time passes, the mean remains approximately $[\text{Fe}/\text{H}] = -0.2$, but the range in metallicity expands as stars from farther afield cross into the annulus. The lack of a significant asymmetry toward lower $[\text{Fe}/\text{H}]$, as observed in the real solar neighborhood, should not be construed as a problem given that the sample is based upon a single generation of stars with a well-defined and

unchanging metallicity distribution.

In contrast, the discontinuity model starts off with a gaussian distribution centered on $[\text{Fe}/\text{H}] = 0.0$ with a dispersion of 0.1 dex. Since we have chosen $R_{GC} = 10$ kpc as the outer boundary, the number of stars from beyond the discontinuity whose initial orbits temporarily place them within the solar neighborhood is negligible. Clearly, this changes as the sample ages to 5 Gyrs and 10 Gyrs. By the intermediate age a significant tail has developed on the metal-poor end of the distribution, extending 0.3 dex below the initial lower edge of the local metallicity distribution. This asymmetry grows stronger in the oldest sample, producing the telltale profile of two gaussians of different weights superimposed. The primary peak in this crudely bimodal distribution remains relatively unaffected near $[\text{Fe}/\text{H}] = 0.0$, while the secondary peak is hidden beneath the combined sample of the outer disk distribution and the metal-poor tail of the inner disk distribution.

The obvious conclusion from this simple comparison is that diffusion has a more dramatic impact upon a discontinuous disk distribution than upon a linear gradient. For the discontinuity, selection of a sample which extends across the break will automatically create an apparent gradient whose slope will depend upon the exact size and location of the baseline. In both cases, gradients derived assuming a linear relation remain relatively unchanged over time. The impact of diffusion is more easily revealed through the metallicity distribution. For the linear case, the sample increases the range in $[\text{Fe}/\text{H}]$ without significantly altering the mean of the distribution. This result is relatively independent of where the annular sample is selected, though it should be remembered that the flow into and out of any annulus is not symmetric because the number of points declines with galactocentric distance. For the discontinuity, the diffusion of stars from the outer disk creates an extended tail of metal-poor stars well below the mean $[\text{Fe}/\text{H}]$ of the inner disk. The size of this tail and its growth rate are strongly dependent upon the exact location of the annular strip defined as the solar neighborhood relative to the boundary of the discontinuity.

3. The Model: Star Clusters

Though the analysis above provides a straightforward indication of the effect of orbital diffusion upon stars, before we construct a more realistic model of the disk with multiple generations of stars, it is important to test if the stellar results are representative of what we expect for star clusters. The issue arises because the presence of the discontinuity appears solely within the cluster sample (Twarog et al. 1997), in part because this is the only stellar sample of statistically significant size which has both reliable distances and metallicities for objects well away from the solar neighborhood in both directions. If our first-order approximation to reality for field stars is correct, diffusion should gradually soften the edges of the discontinuity until it becomes almost indistinguishable from a linear gradient. Moreover, in the solar neighborhood, the sample population should become a mixture of both inner and outer disk objects, making the definition of the break difficult, if not impossible. Why, then, do the clusters define such a sharp break?

Twarog et al. (1997) noted the possibility that the perturbation of an object might be dependent upon its mass; star clusters with masses thousands of times larger than a single star will be negligibly impacted as they orbit the disk in comparison to the effect felt by single field stars. Such an explanation is very dependent upon the origin of the perturbations, something which remains a relative unknown. A simpler solution is provided by the empirical fact that star clusters evaporate over time.

Assume star clusters form with the same efficiency and metallicity distribution as the field stars with galactocentric distance and are perturbed in their orbits to the same degree as the field stars. All things being equal, the data for the star clusters should produce the same results as seen in Figs. 2 and 3. However, as clusters orbit the galactic center, in addition to the perturbations they feel along their orbits, tidal forces combined with the perturbations will also work to remove stars from the gravitational potential well of the cluster. The timescale for the evaporation will depend upon where the cluster is located within the disk, its orbit, the mass of the cluster, and its radius. Fortunately, we do not need a specific

model for the galaxy or a typical cluster to model the evaporation rate; the rate of cluster destruction can be derived empirically by observing the age distribution of a large sample of open clusters over a significant range of the galactic disk. Such an analysis has been carried out by Janes & Phelps (1994), who find that the best fit to the data is a superposition of two exponentials. The galactic models have been rerun with the additional constraint that random points are removed at a rate consistent with the cluster age distribution. The cluster analog of Fig. 2 is presented in Fig. 4. It should be noted that the 10 Gyr sample is the sum of 10 individual runs of the model. For an individual run, the typical number of clusters which remain after 10 Gyrs ranges from 0 to 3, leading to a statistically uninformative plot. By summing over 10 models, one can get a more valuable sense of how the overall distribution is affected by location.

The change in the both samples between 0 and 5 Gyrs highlights the critical point. The reduction in cluster numbers is so dramatic that use of any sample dominated by older clusters leads to an apparent linear gradient, whether the original sample exhibited one or not, as long as there is a statistically significant difference between the mean metallicity of the inner and outer disk. The failure to recognize the discontinuity is not the product of its dissolution by cluster diffusion. As expected, because the diffusion timescale is longer than the cluster evaporation scale, the percentage of clusters that survive long enough to drift into the solar neighborhood is small. This implies that with a representative sample of star clusters covering all ages, a break in the abundance gradient can survive, even though it is unrecognizable among the field stars.

Before embarking on the final phase of this layered approach, a few points regarding the cluster models should be noted. First, the cluster age distribution is not a true representation of what happens to a given generation of clusters. The number of clusters which survive after a time, t , is the convolution of the cluster formation rate over t with the cluster destruction rate, functions which will be dependent upon where the cluster is formed and where its orbit carries it over time. By using all available clusters, Janes & Phelps (1994) have washed out modest variations across the disk,

but the relatively smooth variation in the function with t would seem to indicate that on the global scale, the formation rate of open clusters has not oscillated dramatically over the lifetime of the disk. Second, in contrast with the models, the real disk cluster population exhibits a paucity of open clusters interior to 7 kpc (Janes & Phelps 1994). The majority of clusters older than 2 Gyr are found in the galactic anticenter at respectable distances from the plane, a clear indication that cluster destruction is more efficient inside the solar circle and a probable indication that the scale height of the outer disk is larger than the inner disk. Fortunately, this contradiction between the models and reality does not negate the implications of the models in Fig. 4. If one were to increase the cluster destruction rate in the inner disk and/or lower the rate in the outer disk, the result would be that the numerical asymmetry in the cluster points on either side of $R = 10$ kpc would be reversed, with the likelihood that the majority of the clusters in the 5 and 10 Gyr samples would now be found in the outer disk. What the plots demonstrate, though, is that as long as one has a significant baseline to measure the gradient on one side of the break, the existence of the discontinuity is still detectable. As will be demonstrated more effectively in the cumulative model, while definition of the exact location of the discontinuity depends upon a mapping of clusters between 9 and 11 kpc, proof of the reality of the break is best supplied by extending the baselines of the derived gradients to within $R_{GC} = 7.5$ kpc and beyond 15 kpc.

4. The Model: Cumulative Evolution

Unlike the simple model tested above, the real galactic disk represents a cumulative sample of stars and clusters, formed and evolved over approximately 10 Gyr. During that time, the mean metallicity of the newly formed stars rises as the gas fraction in the disk declines and the products of stellar nucleosynthesis are returned to the interstellar medium to become part of future stellar generations. The rate of star formation and chemical evolution may vary in a complex fashion over both time and location. When coupled with the potential impact of galactic orbital diffusion, the resulting trend of metallicity with galactocentric distance will be a strong function of the age distri-

bution and galactocentric baseline defined by the data sample used to measure it.

In an effort to gain some insight into the plausible effects of the primary processes under discussion, the galactic parameters will be defined as simply as possible. The qualitative impact of deviating from this simple picture will be discussed in the last section. As in Sec. 3, two options for the galactic abundance gradient will be tested, a linear case and a discontinuity. It is assumed that the slope/shape of the disk abundance gradient remains unchanged with time. However, the time dependence of the zero-point of the scale is set such that the value of the metallicity of the disk at $R_{GC} = 8.5$ kpc, the location of the sun, follows an age-metallicity relation approximated by the trend found in Twarog (1980) and corroborated in Meusinger et al. (1991). The star formation rate is assumed to be a constant as a function of time.

The cumulative distribution of 11,000 stars after 10 Gyr of evolution is illustrated for one run of the model in Fig. 5. Fig. 5a (top) shows the distribution of the metallicity with galactocentric distance between $R_{GC} = 5$ and 15 kpc for the linear gradient while Fig. 5b (top) exhibits the comparable data for the discontinuous model. The lower plots in Fig. 5 show the same data on a scale between 7 and 10 kpc. What is readily apparent is that at all galactocentric distance, the range in $[\text{Fe}/\text{H}]$ is large, despite the fact that stars at any given location form with a dispersion of only 0.1 dex. This is due to a combination of both orbital diffusion and the age-metallicity relation. Second, both disks exhibit an apparent gradient. The exact value of the gradient is dependent upon the baseline over which it is measured, particularly for the discontinuous case. For the linear gradient case, the stellar sample shows derived abundance gradients of -0.065 , -0.065 , and -0.058 dex/kpc over the distance range of 0 to 20 kpc, 5 to 15 kpc, and 7 to 10 kpc, respectively. Within the uncertainties, these are identical. For the discontinuous model, the comparable numbers are -0.021 , -0.038 , and -0.029 . Thus, the field stars will produce an observable gradient on any scale, but the size of the gradient will be enhanced in the discontinuous model by the inclusion of a sample skewed in the galactocentric direction of the discontinuity.

A more relevant comparison for the field stars

is that of the metallicity distribution. The distribution of stars as a function of $[\text{Fe}/\text{H}]$ is illustrated in Fig. 6, following a pattern similar to that laid out in Fig. 5. The upper plots include stars at all distances for the two cases, while the lower figures illustrate the respective models using only stars in the $R_{GC} = 7$ to 10 kpc range. The cumulative effects of the differences in the gradient are obvious in the top figures. Because the linear slope is assumed to be constant at -0.07 dex/kpc, if the age-metallicity relation requires a mean $[\text{Fe}/\text{H}]$ near solar over the last 5 Gyrs, the metallicity near the Galactic center approaches $+0.6$, producing a significant fraction of stars with $[\text{Fe}/\text{H}]$ above 0.2. In contrast, this metal-rich tail is virtually absent from the discontinuous model. Note, however, that the obvious differences between these two cases disappear if we restrict our data to field stars in the solar neighborhood, as seen in the lower plots of Fig. 6. The metal-rich stars of the inner galaxy are rarely found in the galactic suburbs, eliminating the high-metallicity tail. However, the low-metallicity portion of the distribution is found in both samples, the product of the age-metallicity relation and modest orbital diffusion in both cases.

In contrast, the trends for star clusters are much cleaner, as illustrated in Fig. 7 for the usual cases. In both instances, the observed gradient for the clusters has the same well-defined pattern as adopted for each model. The reason is that the samples are invariably dominated by the clusters formed over the last 2 Gyr. A handful of clusters manage to survive beyond 5 Gyr, producing an occasional low metallicity outlier at a given distance due to the lower $[\text{Fe}/\text{H}]$ among older clusters and a possibility of significant drift. For the most part, the sample still reflects the abundance gradient of the most recently formed stars. If the slope/shape of the gradient of newly formed stars remains relatively unchanged over time, the cluster sample offers the best hope of detailing the gradient over large galactocentric distance, followed closely by high mass stars with readily defined distance moduli, e.g., Cepheids (Fry & Carney 1997). One can also attempt to search for time variation of the gradient by sorting the cluster sample by age but, as noted in the last section, it will be statistically difficult to derive any pattern except the simplest linear slope through the data points, irrespective

of the detailed shape.

We return now to the issue which initially motivated this discussion, the dispersion in the age-metallicity relation. If diffusion does not occur, and the intrinsic dispersion of newly forming stars remains a constant with age of the disk, and we can measure stellar ages with perfect accuracy, the metallicity dispersion as a function of age should remain a constant defined by the dispersion found in the solar annulus at the time of formation of the disk. Though the dispersion in $[\text{Fe}/\text{H}]$ is assumed to be 0.1 dex, the changing $[\text{Fe}/\text{H}]$ across the solar annulus raises the initial value for the linear gradient model to 0.114 dex. Any change in this value is a product of diffusion. The calculated age-metallicity relation for the cumulative sample is shown in Fig. 8 for the (a) linear model and (b) the discontinuity. The error bars around each point show the size of the dispersion in $[\text{Fe}/\text{H}]$ at a given age. The trends for both models are similar: for samples of increasing age between 0 and 2 Gyrs, the dispersion grows by about 50 % before leveling off for the discontinuous model while increasing slightly for the linear case for older stellar samples. The typical model dispersion in $[\text{Fe}/\text{H}]$ among stars from the old disk is between 0.15 and 0.17 dex. Taking into account the additional impact of uncertainties in age and metallicity, this dispersion is compatible with the results of Twarog (1980), Meusinger et al. (1991), and Rocha-Pinto et al. (2000), but inadequate to meet the demands of Edvardsson et al. (1993), emphasizing again the likelihood that the last sample is affected by a selection bias which creates an anomalously large dispersion.

5. Summary and Conclusions

The primary goal of this investigation has been a differential comparison of the effects of orbital diffusion triggered by perturbations within the galactic disk on a sample of field stars and a sample of star clusters. Whether all stars initially form within clusters or the majority of stars form within the field and small groups (Adams & Myers 2001), it is assumed that both populations follow the same metallicity trend with galactocentric distance and age. The specific galactic gradients modelled include a simple linear trend with a constant slope over the entire disk at all ages

and a discontinuous trend, essentially flat on either side of a 0.3 dex drop near $R_{GC} = 10$ kpc. For the cumulative evolution of the disk over time, clusters are destroyed at a rate consistent with the observed age distribution of clusters within a few kpc of the sun.

In the single generation comparison, the effect of diffusion for both gradients is to dramatically increase the dispersion in $[\text{Fe}/\text{H}]$ among the field stars within the solar neighborhood, arbitrarily defined as a 3 kpc-wide annulus centered on the sun. Though the slope of the gradient remains essentially unchanged for the linear case, diffusion from beyond the solar circle washes out the edge of the discontinuity and creates an artificial gradient among the stars in the solar neighborhood. The exact size of the gradient depends upon the galactocentric baseline used. For the cluster sample, the dominant effect is cluster evaporation. If all clusters survived, the expectation is that the trends noted for the field stars would be reproduced by the clusters. However, cluster evaporation reduces the sample by such an extreme amount that the probability of a cluster diffusing across the boundary and surviving for more than a few Gyrs is extremely small.

Despite the artificially restricted nature of the chemical evolution of the cumulative disk, the patterns noted above are enhanced and the differences between the models diminished. For the field stars, the time variation of $[\text{Fe}/\text{H}]$ expands the range of metallicity included in the final sample well beyond that imposed by the abundance gradient. Though a significant difference in the $[\text{Fe}/\text{H}]$ range is apparent over the entire disk because of the need to extrapolate the linear gradient to high $[\text{Fe}/\text{H}]$ near the galactic center, within the solar circle typical of the pool used to select field stars for analysis, the metallicity distributions of the two gradient models become virtually indistinguishable and bear an amazing resemblance to the real field star distribution. Both models produce an apparent gradient; the diffusion destroys any possibility of identifying the discontinuity among the field stars.

In sharp contrast, the cluster sample provides an ideal means of distinguishing between the two cases. The fundamental reason is that cluster evaporation guarantees that any significant, random sample of clusters will be dominated by the

clusters formed over the last 2 Gyrs. On this timescale, the effects of diffusion are mild and the sample retains the gradient which existed at the time of formation. Unless the gradient undergoes serious evolution on a 2 Gyr timescale, open clusters offer the best stellar option for detailing the recent galactic abundance gradient over large galactocentric distances. One could attempt to identify the gradient in the past by isolating older open clusters. However, what the models clearly demonstrate is that, like the field stars, even if a significant statistical sample can be collected, diffusion will wipe out any discontinuity, making a distinction between a real linear gradient and a diffused discontinuity difficult, if not impossible. Moreover, if one uses the traditional sample of fewer than 20 old disk clusters, any attempt to distinguish between the two cases or even evaluate a potential variation in the gradient with time is statistically futile. One will basically define the minimal case, a linear relation, whose slope may bear little relation to the true trend with the galaxy at the time of formation. Thus, claims regarding the consistency of the abundance gradient found among old clusters with that defined by the young disk are more likely an indication of the inadequacy of the sample than a measure of a fundamental characteristic of galactic evolution.

Given the above, two issues come to mind. First, are the models representative of the real disk or are the features merely artifacts of an overly simplistic description of the disk? In an absolute sense, the models undoubtedly produce only a mild reflection of reality, particularly in the cumulative case. If the source of the perturbations were known, one could vary the size of the effect with time and galactocentric distance. It is improbable that the abundance gradient maintains a constant shape over 10 Gyrs worth of evolution; chemical evolution is likely to occur at a higher pace closer to the galactic center, describing an age-metallicity relation which differs from that found near the sun. Unfortunately, attempts to detect an age trend by defining the gradient using classes of objects with different mean ages (old clusters vs. young, HII regions vs. planetary nebulae) are often swamped by the statistical uncertainty in defining the slope within each category. Additionally, chemical evolution models may be constructed which produce abundance gradients

which grow steeper or shallower with time, making it difficult to know which trend is appropriate for our model (Hou et al. 2000). As a general rule, the effect of diffusion is to make a gradient slightly shallower over time while smoothing out discontinuities over galactocentric distance. If the abundance gradient grows shallower over time because of the delayed increase in the metallicity of the outer disk relative to the inner, the primary impact will be to smooth the transition between the flatter gradient in the inner disk and the steeper gradient away from the center while making the outer gradient somewhat weaker.

More directly, we know from observation that the dissolution of star clusters occurs at a faster rate within $R_{GC} = 7.5$ kpc. The model cluster distribution maintains the same approximate profile with galactocentric distance over 10 Gyrs, leading to a final sample heavily weighted toward the center of the Galaxy. With the exception of the last point, which is predominantly an observational constraint addressed below, most of the modelling issues which arise dominate on scales beyond the solar circle. Over the primary galactocentric range of interest, 7 to 10 kpc, the errors in the absolute sense do not significantly alter the differential comparisons of the models. There is little question that the net effect of diffusion is to destroy the detailed structural information content of the field star sample over time. The bigger the adopted diffusion coefficient and/or the larger the time frame, the more likely it is that one will be unable to distinguish between a simple linear gradient or a more complex gradient structure. Despite this uncertainty, a derived linear gradient among the field stars implies a difference in $[\text{Fe}/\text{H}]$ of some form between the inner disk and the outer disk.

Second, if the field stars lose their ability to inform us about the detailed structure of the disk over time, can the cluster population resolve the issue? Because one has the capability of reliably identifying clusters by age and isolating the younger portion of the distribution, the best approach to looking for structure in the galactic abundance gradient lies with the cluster population. By default, a random sample of clusters will invariably be dominated by the younger portion of the disk, which explains why the sample of Twarog et al. (1997) was capable of identifying the dis-

continuity. More important is that fact that if a sudden transition in slope is found for $[\text{Fe}/\text{H}]$ as a function of galactocentric distance using a statistically significant sample of younger clusters, it is highly probable that it is real. (A similar conclusion could be postulated for field stars if one could reliably define age and metallicity for a respectable sample of field stars out to a distance of 2.5 kpc on either side of the solar circle.) Adjustments to account for the simplistic assumptions of the model noted above tend to have, at worst, a weak impact on the local gradient or, if included, would weaken the appearance of a discontinuity. As an example, if the star formation rate isn't a constant over time but was higher in the earlier history of the disk, this would increase the probability of stars of lower $[\text{Fe}/\text{H}]$ diffusing into the solar circle and expanding the observed range of $[\text{Fe}/\text{H}]$ locally, while placing more metal-rich stars and clusters beyond $R_{GC} = 10$ kpc, turning a potential sharp gradient into a shallow one. Even for the clusters, the fraction that survive will increase and weaken the feature relative to the current sample. This will have no impact, however, if one uses only clusters below 2 Gyrs in age. While it is a straightforward task to destroy a sharp feature through time evolution or observational error, it is equally difficult to create a sharp transition through such means in a sample where the feature was initially absent.

We close by noting one additional test of the reality of the feature that is within the limits of cluster observation. A common criticism of the discontinuity is the claim that the slope is a product of too small a baseline, particularly for the sample on the solar side of the break. The majority of the clusters studied (~ 62) by Twarog et al. (1997) lie between 7.5 and 10 kpc while the small sample beyond the break (14) ranges over 5 kpc. If the break is real and there is little or no gradient on either side, the critical test is to study the gradient in the two zones independently, i.e., expand the sample inside 10 kpc to a distance closer to $R_{GC} = 2.5$ kpc, and add more clusters to the 5 kpc range beyond the break. Any analysis which includes the zone between 8 and 12 kpc will be guaranteed exhibit a measurable linear slope, whether that is the correct form or not.

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Fig. 1.— Velocity dispersions for the individual velocity components and the total velocity as a function of time for the model with a velocity-dependent diffusion coefficient.

Fig. 2.— Evolution of the abundance gradient for (a) the linear case and (b) the discontinuity. The number after the letter gives the age in Gyrs. Note that the $[\text{Fe}/\text{H}]$ ranges differ for the two cases.

Fig. 3.— Metallicity distributions for stars between $R_{GC} = 7$ and 10 kpc for the same models detailed in Fig. 2.

Fig. 4.— Same as Fig. 2 but for open clusters. The 10 Gyr sample is the sum of 10 model runs.

Fig. 5.— Metallicity distribution with galactocentric distance for (a) the linear gradient and (b) the discontinuous disk after 10 Gyr of cumulative evolution. The lower figures show an expanded view of the region defined as the solar neighborhood.

Fig. 6.— Metallicity histograms for the same cases as in Fig. 5, covering the entire range of galactocentric distance in the upper figures and the solar neighborhood in the lower figures.

Fig. 7.— Metallicity trend with galactocentric distance for star clusters under the assumption of (a) a linear gradient and (b) a discontinuous disk. Sample includes all clusters which form and survive over the 10 Gyr lifetime of the disk.

Fig. 8.— Age-metallicity relations for the models with (a) a discontinuous gradient and (b) a linear gradient for stars in the solar neighborhood.